Please check the examination details below before entering your candidate information

Candidate surname	Other names

Pearson Edexcel
International
Advanced Level

Centre Number ☐☐☐☐☐ Candidate Number ☐☐☐☐

Wednesday 21 October 2020

Morning (Time: 1 hour 30 minutes) Paper Reference **WMA12/01**

Mathematics

International Advanced Subsidiary/Advanced Level
Pure Mathematics P2

You must have:
Mathematical Formulae and Statistical Tables (Lilac), calculator

Total Marks

Candidates may use any calculator permitted by Pearson regulations. Calculators must not have the facility for symbolic algebra manipulation, differentiation and integration, or have retrievable mathematical formulae stored in them.

Instructions

- Use **black** ink or ball-point pen.
- If pencil is used for diagrams/sketches/graphs it must be dark (HB or B).
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions and ensure that your answers to parts of questions are clearly labelled.
- Answer the questions in the spaces provided – *there may be more space than you need.*
- You should show sufficient working to make your methods clear. Answers without working may not gain full credit.
- Inexact answers should be given to three significant figures unless otherwise stated.

Information

- A booklet 'Mathematical Formulae and Statistical Tables' is provided.
- There are 9 questions in this question paper. The total mark for this paper is 75.
- The marks for **each** question are shown in brackets – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Try to answer every question.
- Check your answers if you have time at the end.
- If you change your mind about an answer, cross it out and put your new answer and any working underneath.

Turn over ▶

1. (a) Find the first 4 terms, in ascending powers of x, of the binomial expansion of

$$\left(2 - \frac{x}{4}\right)^{10}$$

giving each term in its simplest form.

(4)

(b) Hence find the constant term in the series expansion of

$$\left(3 - \frac{1}{x}\right)^{2}\left(2 - \frac{x}{4}\right)^{10}$$

(3)

a. BINOMIAL EXPANSION FORMULA:

$$(a+bx)^n = a^n + ({}^nC_1)a^{n-1}(bx)^1 + ({}^nC_2)a^{n-2}(bx)^2 + . + ({}^nC_{n-1})a^1(bx)^{n-1} + (bx)^n$$

where $^nC_r = \binom{n}{1} = \frac{n!}{r!(n-r)!}$

$$\left(2-\frac{x}{4}\right)^{10} = 2^{10} + ({}^{10}C_1)(2^{10-1})\left(-\frac{x}{4}\right)^1 + ({}^{10}C_2)(2^{10-2})\left(-\frac{x}{4}\right)^2 + ({}^{10}C_3)(2^{10-3})\left(-\frac{x}{4}\right)^3$$ ✓M1

$$= 1024 + 10 \times 512 \times -\frac{x}{4} + 45 \times 256 \times \frac{x^2}{16} + 120 \times 128 \times -\frac{x^3}{64}$$

$$= 1024 - 1280x + 720x^2 - 240x^3$$

$$\boxed{= 1024 - 1280x + 720x^2 - 240x^3}$$ ✓B1 ✓A1 ✓A1

b $\left(3 - \frac{1}{x}\right)^2 = 9 - \frac{6}{x} + \frac{1}{x^2}$ ✓M1

Notice that the constant term can be made by multiplying corresponding terms in the expansions (constant × constant, x term × x term, etc) as the x cancels out

$$\left(9 - \frac{6}{x} + \frac{1}{x^2}\right)(1024 - 1280x + 720x^2 - 240x^3) = 9 \times 1024 - \frac{6}{x} \times -1280x + \frac{1}{x^2} \times 720x^2$$ ✓M1

$$= \boxed{17616}$$ ✓A1

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Question 1 continued

Turn over ▶

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Question 1 continued

Leave blank

Question 1 continued

Q1

(Total 7 marks)

2.

$$y = \frac{2^x}{\sqrt{(5x^2 + 3)}}$$

(a) Complete the table below, giving the values of y to 3 decimal places.

x	-0.25	0	0.25	0.5	0.75
y	0.462	0.577	0.653	0.686	0.698

(1)

(b) Use the trapezium rule, with all the values of y from the completed table, to find an approximate value for

$$\int_{-0.25}^{0.75} \frac{2^x}{\sqrt{(5x^2 + 3)}}\,dx$$

(3)

a. $y = \frac{2^x}{\sqrt{(5x^2 + 3)}}$

$x = 0$

$y = \frac{2^0}{\sqrt{(5(0)^2 + 3)}}$

$y = 0.577$ (3dp)

$x = 0.5$

$y = \frac{2^{0.5}}{\sqrt{(5(0.5)^2 + 3)}}$

$y = 0.686$ (3dp) ✓B1

b. TRAPEZIUM RULE

$\int_a^b y\,dx = \frac{1}{2}h\left[(y_0 + y_n) + 2(\underbrace{y_1 + y_2 + \ldots + y_{n-1}})\right]$, where $h = \frac{b-a}{n}$

represents the width of each strip

height of the first and last strip respectively

heights of strips in between the first and last

number of strips

$h = \frac{0.75 - -0.25}{4}$

There are 5 coordinates, so $5 - 1 = 4$ strips used

$h = 0.25$ ✓B1

$\int_{-0.25}^{0.75} \frac{2^x}{\sqrt{(5x^2 + 3)}} = \frac{1}{2} \times 0.25 \times (0.462 + 0.698 + 2(0.577 + 0.653 + 0.686))$ ✓M1

$= \boxed{0.624 \text{ (3dp)}}$ ✓A1

Leave blank

Question 2 continued

Q2

(Total 4 marks)

3. $$f(x) = ax^3 - x^2 + bx + 4$$

where a and b are constants.

When $f(x)$ is divided by $(x + 4)$, the remainder is -108

(a) Use the remainder theorem to show that

$$16a + b = 24$$

(2)

Given also that $(2x - 1)$ is a factor of $f(x)$,

(b) find the value of a and the value of b.

(3)

(c) Find $f'(x)$.

(1)

(d) Hence find the exact coordinates of the stationary points of the curve with equation $y = f(x)$.

(4)

a. REMAINDER THEOREM

When a polynomial, $f(x)$, is divided by $(x-a)$, the remainder is $f(a)$

Dividing $f(x)$ by $(x+4)$ means our a value is -4, as $x+4 = x-(-4)$

Sub in $x=-4$

$f(-4) = a(-4)^3 - (-4)^2 + b(-4) + 4$ ✓M1

$-108 = -64a - 16 - 4b + 4$

$-108 = -64a - 4b - 12$

$64a + 4b = 96$

$16a + b = 24$ ①

$\boxed{16a + b = 24}$ ✓A1*

b. FACTOR THEOREM: $(ax-b)$ is a factor of $f(x)$ if $f(\frac{b}{a}) = 0$ $(ax-b=0 \Rightarrow x = \frac{b}{a})$

As stated by the factor theorem above, since $(2x-1)$ is a factor, $f(\frac{1}{2}) = 0$

Sub in $x = \frac{1}{2}$

$f(\frac{1}{2}) = a(\frac{1}{2})^3 - (\frac{1}{2})^2 + b(\frac{1}{2}) + 4$

$0 = \frac{1}{8}a - \frac{1}{4} + \frac{1}{2}b + 4$

x8 $\frac{1}{8}a + \frac{1}{2}b = -\frac{15}{4}$

$a + 4b = -30$ ② ✓M1

Question 3 continued

Solve ① and ② simultaneously

16× ② - ①

$16a + 64b = -480$ ✓M1

$-(16a + b = 24)$

$63b = -504$

$b = -8$

Sub $b = -8$ into ①

$16a + (-8) = 24$

$16a = 32$

$a = 2$

$\boxed{\therefore a = 2, b = -8}$ ✓A1

c. Using the values of a and b from part (b)

$f(x) = 2x^3 - x^2 - 8x + 4$

$f'(x) = 6x^2 - 2x - 8$

$\boxed{f'(x) = 6x^2 - 2x - 8}$ ✓B1ft

d. At stationary points, $f'(x) = 0$

$f'(x) = 6x^2 - 2x - 8$

$6x^2 - 2x - 8 = 0$ ↓Factorise

$(3x-4)(x+1) = 0$ ✓M1

$3x - 4 = 0 \qquad x + 1 = 0$

$x = \frac{4}{3} \qquad x = -1$

Sub x values into f(x) to find y values

$f(\frac{4}{3}) = 2(\frac{4}{3})^3 - (\frac{4}{3})^2 - 8(\frac{4}{3}) + 4 \qquad f(-1) = 2(-1)^3 - (-1)^2 - 8(-1) + 4$ ✓M1

$f(\frac{4}{3}) = -\frac{100}{27} \qquad f(-1) = 9$

$\boxed{(\frac{4}{3}, -\frac{100}{27}), (-1, 9)}$ ✓A1

Leave blank

Question 3 continued

Question 3 continued

Q3

(Total 10 marks)

4. The points P and Q have coordinates $(-11, 6)$ and $(-3, 12)$ respectively.

Given that PQ is a diameter of the circle C,

(a) (i) find the coordinates of the centre of C,

(ii) find the radius of C.

(4)

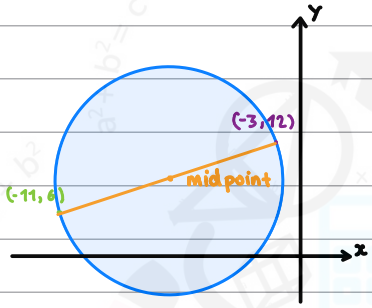
(b) Hence find an equation of C.

(2)

(c) Find an equation of the tangent to C at the point Q giving your answer in the form $ax + by + c = 0$ where a, b and c are integers to be found.

(3)

ai. Since PQ is the diameter of the circle C, the centre of C must be its midpoint and its length halved is its radius.



Centre:

$$\text{midpoint} = \left(\frac{-11 + -3}{2}, \frac{6+12}{2}\right)$$

$$= (-7, 9)$$

$$= \boxed{(-7, 9)}$$ ✓B1 ✓B1

aii. Find distance between centre and point on circle

DISTANCE BETWEEN TWO POINTS FORMULA

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$

$$\text{radius} = \sqrt{(-7 - -11)^2 + (9-6)^2}$$ ✓M1

$$= 5$$

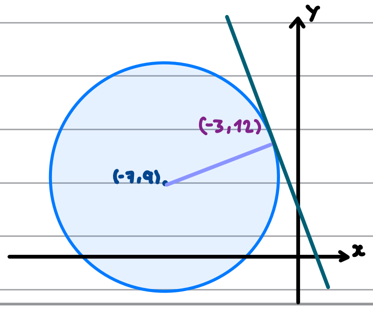
$$= \boxed{5}$$ ✓A1

b. Centre is $(-7, 9)$ and radius is 5

$$\therefore (x - -7)^2 + (y-9)^2 = 5^2$$

$$\boxed{(x+7)^2 + (y-9)^2 = 25}$$ ✓M1 ✓A1

c.



Find the gradient of line connecting point Q and centre

$$\text{gradient} = \frac{12-9}{-3 - -7}$$

$$= \frac{3}{4}$$ ✓M1

Question 4 continued

Gradient of tangent = negative reciprocal of line connecting point Q and centre

gradient of tangent $= -\frac{1}{3/4}$

$= -\frac{4}{3}$

Use the point-slope equation of line to find equation of tangent

POINT- SLOPE EQUATION OF LINE

$y - y_1 = m(x - x_1)$

$y - 12 = -\frac{4}{3}(x - -3)$ ✓M1

$y - 12 = -\frac{4}{3}(x + 3)$ Multiply by 3 to remove the fraction

$3y - 36 = -4x - 12$

$\boxed{4x + 3y - 24 = 0}$ ✓A1

Leave blank

Question 4 continued

Leave blank

Question 4 continued

Q4

(Total 9 marks)

5. Ben is saving for the deposit for a house over a period of 60 months.

Ben saves £100 in the first month and in each subsequent month, he saves £5 more than the previous month, so that he saves £105 in the second month, £110 in the third month, and so on, forming an arithmetic sequence.

(a) Find the amount Ben saves in the 40th month. **(2)**

(b) Find the total amount Ben saves over the 60-month period. **(3)**

Lina is also saving for a deposit for a house.

Lina saves £600 in the first month and in each subsequent month, she saves £10 less than the previous month, so that she saves £590 in the second month, £580 in the third month, and so on, forming an arithmetic sequence.

Given that, after *n* months, Lina will have saved exactly £18 200 for her deposit,

(c) form an equation in *n* and show that it can be written as

$$n^2 - 121n + 3640 = 0$$

(3)

(d) Solve the equation in part (c). **(2)**

(e) State, with a reason, which of the solutions to the equation in part (c) is **not** a sensible value for *n*. **(1)**

a. Recall the formula for the n^{th} term of an arithmetic sequence

NTH TERM OF ARITHMETIC SEQUENCE

$a_n = a + (n-1)d$

First term, $a = 100$

Common difference, $d = 5$

$a_{40} = 100 + (40-1) \times 5$ ✓M1

$= \boxed{£295}$ ✓A1

b. ARITHMETIC SERIES

$S_n = \frac{n}{2}(2a + (n-1)d)$

$S_{60} = \frac{60}{2}(2 \times 100 + (60-1) \times 5)$ ✓M1

$= 30(200 + 59 \times 5)$ ✓A1

$= \boxed{£14850}$ ✓A1

Question 5 continued

c. First term, $a = 600$

Common difference, $d = -10$

$S_n = \frac{n}{2}(2 \times 600 + (n-1) \times -10)$

$18200 = \frac{n}{2}(1200 - 10n + 10)$ ✓M1

$= \frac{n}{2}(1210 - 10n)$

$18200 = 605n - 5n^2$

$5n^2 - 605n + 18200 = 0$ } ÷5

$n^2 - 121n + 3640 = 0$ ✓A1

d. $n^2 - 121n + 3640 = 0$

$(n-56)(n-65) = 0$ ✓M1

$n - 56 = 0 \quad n - 65 = 0$

$n = 56 \quad n = 65$ ✓A1

e. $n = 65$ is not sensible because:

· The money has already been saved after 56 months

· At the 65th month, Lisa will save $600 + (65-1)(-10) = -40$ which is nonsensical

} Either reason is fine for ✓B1

Question 5 continued

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Question 5 continued

Q5

(Total 11 marks)

6.

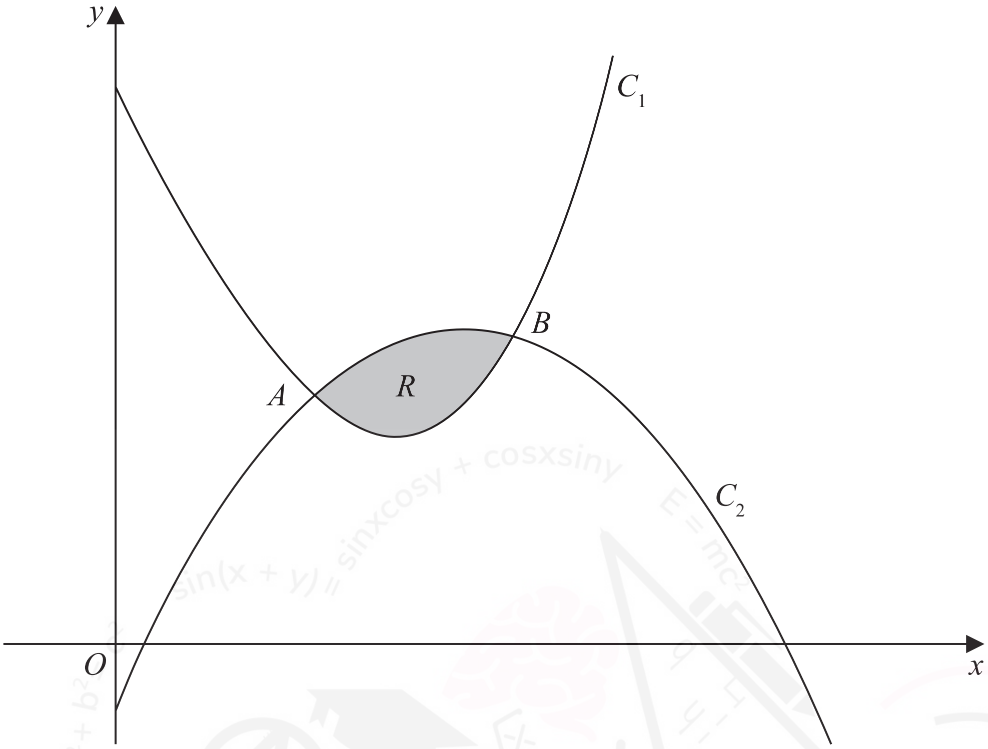


Figure 1

Figure 1 shows a sketch of part of the curves C_1 and C_2 with equations

$$C_1 : y = x^3 - 6x + 9 \qquad x \geqslant 0$$
$$C_2 : y = -2x^2 + 7x - 1 \qquad x \geqslant 0$$

The curves C_1 and C_2 intersect at the points A and B as shown in Figure 1.

The point A has coordinates (1, 4).

Using algebra and showing all steps of your working,

(a) find the coordinates of the point B.

(4)

The finite region R, shown shaded in Figure 1, is bounded by C_1 and C_2

(b) Use algebraic integration to find the exact area of R.

(5)

a. $x^3 - 6x + 9 = -2x^2 + 7x - 1$ ✓M1

$x^3 + 2x^2 - 13x + 10 = 0$ ✓A1

Since point A(1,4) is one of the points of intersection of C_1 and C_2, $x=1$ is a solution to the cubic polynomial above, so $(x-1)$ is a factor.

Use algebraic division to find the quadratic factor we can solve.

Question 6 continued

$$x^2 + 3x - 10$$

$$x-1 \overline{\big)\, x^3 + 2x^2 - 13x + 10}$$

$$\underline{-(x^3 - x^2)}$$

$$3x^2 - 13x$$

$$\underline{-(3x^2 - 3x)}$$

$$-10x + 10$$

$$\underline{-(-10x + 10)}$$

$$0 \quad 0$$

$\therefore x^3 + 2x^2 - 13x + 10 = (x-1)(x^2 + 3x - 10)$ ✓M1

Solve $x^2 + 3x + 10 = 0$

$x^2 + 3x - 10 = 0$

QUADRATIC FORMULA

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-3 \pm \sqrt{3^2 - 4 \times 1 \times -10}}{2 \times 1}$$

$x = -5$, $x = 2$ ← B has to have a positive x-coordinate from figure 1

Sub in $x=2$ into C_1 to find the y-coordinate of B.

$(2)^3 - 6(2) + 9 = 8 - 12 + 9$

$= 5$

$\therefore B(2,5)$ ✓A1

b. **FORMULA FOR AREA BETWEEN TWO CURVES**

$$\text{area} = \int_a^b y_{TOP} - y_{BOTTOM} \, dx$$

Between A and B, C_2 is above C_1, so:

Area between C_1 and C_2 $= \int_A^B C_2 - C_1 \, dx$

$= \int_1^2 -2x^2 + 7x - 1 - (x^3 - 6x + 9) \, dx$

$= \int_1^2 -2x^2 + 7x - 1 - x^3 + 6x - 9 \, dx$

$= \int_1^2 -x^3 - 2x^2 + 13x - 10 \, dx$

$= \left[-\frac{1}{4}x^4 - \frac{2}{3}x^3 + \frac{13}{2}x^2 - 10x \right]_1^2$ ✓M1 ✓dM1 ✓A1

$= -\frac{1}{4}(2)^4 - \frac{2}{3}(2)^3 + \frac{13}{2}(2)^2 - 10(2) - \left[-\frac{1}{4}(1)^4 - \frac{2}{3}(1)^3 + \frac{13}{2}(1)^2 - 10(1) \right]$ ✓ddM1

$= -\frac{10}{3} - \left[-\frac{53}{12} \right] = \boxed{\frac{13}{12}}$ ✓A1

Leave blank

Question 6 continued

Question 6 continued

(Total 9 marks)

Q6

7. (i) Show that

$$\tan\theta + \frac{1}{\tan\theta} \equiv \frac{1}{\sin\theta\cos\theta} \qquad \theta \neq \frac{n\pi}{2} \qquad n \in \mathbb{Z}$$

(3)

(ii) Solve, for $0 \leqslant x < 90°$, the equation

$$3\cos^2(2x + 10°) = 1$$

giving your answers in degrees to one decimal place.

(*Solutions based entirely on graphical or numerical methods are not acceptable.*)

(4)

i. ① $\boxed{\tan\theta = \frac{\sin\theta}{\cos\theta}}$ ② $\boxed{\sin^2\theta + \cos^2\theta = 1}$

$$\tan\theta + \frac{1}{\tan\theta} = \frac{\sin\theta}{\cos\theta} + \frac{1}{\frac{\sin\theta}{\cos\theta}} \quad ①$$

$$= \frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}$$

$$= \frac{\sin^2\theta + \cos^2\theta}{\cos\theta\sin\theta}$$

$$= \frac{1}{\cos\theta\sin\theta} \quad ②$$

$$= \boxed{\frac{1}{\sin\theta\cos\theta}}$$

ii. $3\cos^2(2x+10°) = 1$

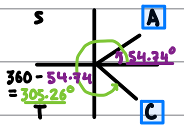
$\left.\begin{aligned} \cos^2(2x+10°) &= \frac{1}{3} \\ \cos(2x+10°) &= \pm\sqrt{\frac{1}{3}} \end{aligned}\right\}$ M1

The range given, $0° \leqslant x < 90°$, can be transformed to $10° \leqslant 2x + 10 < 190°$

$\cos(2x+10°) = \sqrt{\frac{1}{3}}$ ← Value of cos is +ve, so we want the 1st and 4th quadrant of CAST diagram

✓M1 $\left\{\begin{aligned} 2x+10 &= \cos^{-1}\left(\sqrt{\frac{1}{3}}\right) \\ 2x+10 &= 54.74°, 305.26° \\ 2x &= 44.74° \end{aligned}\right.$

$x = 22.367°$



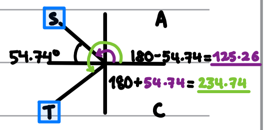
$\cos(2x+10°) = -\sqrt{\frac{1}{3}}$ ← -ve, so we want 2nd and 3rd quadrants of CAST diagram

$X = \cos^{-1}\left(\sqrt{\frac{1}{3}}\right)$

$X = 54.74$

$2x+10 = 125.26°, 234.74°$

$x = 57.632°$



$\boxed{22.4°, 57.6°}$ ✓A1 (for 1) ✓A1 (for both)

Question 7 continued

Q7

(Total 7 marks)

8. A geometric series has first term a and common ratio r.

(a) Prove that the sum of the first n terms of this series is given by

$$S_n = \frac{a(1-r^n)}{1-r}$$

(3)

The second term of a geometric series is –320 and the fifth term is $\frac{512}{25}$

(b) Find the value of the common ratio.

(2)

(c) Hence find the sum of the first 13 terms of the series, giving your answer to 2 decimal places.

(3)

a. To prove, the first step is to write out the terms of the series

$S_n = a + ar + ar^2 + \ldots + ar^{n-1}$

✓M1

Multiply each term in the series by the common ratio, r

$rS_n = ar + ar^2 + ar^3 + \cdots + ar^n$

If we subtract rS_n from S_n, we can see the terms in the middle of the series cancel out

$S_n : a + \cancel{ar} + \cancel{ar^2} + \cancel{\cdots} + \cancel{ar^{n-1}}$

$- \; rS_n : \cancel{ar} + \cancel{ar^2} + \cancel{ar^3} + \cancel{\cdots} + ar^n$

$S_n - rS_n : a - ar^n$

Factorising and making S_n the subject

$S_n - rS_n = a - ar^n$ ✓A1

$S_n(1-r) = a(1-r^n)$

$\boxed{S_n = \frac{a(1-r^n)}{1-r}}$ ✓A1

b. Second term: $ar = -320$

Fifth term: $ar^4 = \frac{512}{25}$

$\frac{ar^4}{ar} = \frac{\frac{512}{25}}{-320}$

$r^3 = -\frac{8}{125}$

$r = \sqrt[3]{-\frac{8}{125}}$

✓M1

$r = -\frac{2}{5}$

$\boxed{r = -\frac{2}{5}}$ ✓A1

Question 8 continued

c. We have the common ratio and the 2nd term, so we can work out the first term

Second term: $ar = -320$

Common ratio: $r = -\frac{2}{5}$

First term: $\frac{ar}{r} = \frac{-320}{-\frac{2}{5}}$

$= 800$ ✓M1

Use the sum of nth term of geometric series formula as given in part a.

GEOMETRIC SERIES

$S_n = \frac{a(1-r^n)}{1-r}$

$S_{13} = \frac{800(1-(-\frac{2}{5})^{13})}{1-(-\frac{2}{5})}$ ✓M1

$= \boxed{571.43\ (2dp)}$ ✓A1

Leave blank

Question 8 continued

Leave blank

Question 8 continued

(Total 8 marks)

Q8

9. (i) Find the exact value of x for which

$$\log_3(x + 5) - 4 = \log_3(2x - 1)$$

(4)

(ii) Given that

$$3^{y+3} \times 2^{1-2y} = 108$$

(a) show that

$$0.75^y = 2$$

(4)

(b) Hence find the value of y, giving your answer to 3 decimal places.

(2)

i. ① $\boxed{\log\left(\frac{a}{b}\right) = \log(a) - \log(b)}$

Rewrite 4 in logs so we can combine the logs together.

$\log_3(x+5) - 4 = \log_3(2x-1)$ ✓B1

$\log_3(x+5) - \log_3(3^4) = \log_3(2x-1)$

$\log_3(x+5) - \log_3(81) = \log_3(2x-1)$

① $\log_3\left(\frac{x+5}{81}\right) = \log_3(2x-1)$ ✓M1

$\frac{x+5}{81} = 2x-1$ ✓A1

$x+5 = 162x - 81$

$161x = 86$

$\boxed{x = \frac{86}{161}}$

iia. ① $\boxed{a^{b+c} = a^b \times a^c}$

$3^{y+3} \times 2^{1-2y} = 108$

① $3^3 \times 3^y \times 2^1 \times 2^{-2y} = 108$ ✓B1 ✓M1

$27 \times 2 \times 3^y \times 2^{-2y} = 108$

$54 \times 3^y \times 2^{-2y} = 108$

$3^y \times 2^{-2y} = \frac{108}{54}$

$\frac{3^y}{2^{2y}} = 2$ ✓M1

$\frac{3^y}{4^y} = 2$

Question 9 continued

$\left(\frac{3}{4}\right)^{Y} = 2$

$(0.75)^{Y} = 2$

$\boxed{(0.75)^{Y} = 2}$ ✓A1

iib. $\boxed{\log(a^{b}) = b\log(a)}$

$0.75^{Y} = 2$ log both sides, can alternatively log both sides to the base of 0.75 to get Y directly

$\log 0.75^{Y} = \log 2$

$Y\log 0.75 = \log 2$

$Y = \frac{\log 2}{\log 0.75}$ ✓M1

$\boxed{Y = -2.409 \text{ (3dp)}}$ ✓A1

Question 9 continued

Q9

(Total 10 marks)

TOTAL FOR PAPER IS 75 MARKS

END